On Some New Results in Discrete Tomography

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Discrete tomography deals with the reconstruction of finite sets from knowledge about their interaction with certain query sets. The most prominent example is that of the reconstruction of a finite subset F of $\mathbb{Z}^d$ from its X-rays (i.e., line sums) in a small positive integer number m of directions. Applications of discrete tomography include quality control in semiconductor industry, image processing, scheduling, and statistical data security. The reconstruction task is an ill-posed discrete inverse problem, depicting (suitable variants of) all three Hadamard criteria for ill-posedness.

After a short introduction to the field of discrete tomography, the first part of the talk addresses the following questions. Does discrete tomography have the power of error correction? Can noise be compensated by taking more X-ray images, and, if so, what is the quantitative effect of taking one more X-ray? Our main theoretical result gives the first nontrivial unconditioned (and best possible) stability result. On the algorithmic side we show that while there always is a certain inherent stability, the possibility of making (worst-case) efficient use of it is rather limited.

The second part of the talk deals with the discrete tomography of quasicrystals that live on finitely generated $\mathcal{Z}$-modules in some $\mathcal{R}^s$. Focussing on aspects in which the discrete tomography of quasicrystals differs from that in the classical lattice case, we solve a basic decomposition problem for the discrete tomography of quasicystals. More generally, we study the problem of existence of pseudodiophantine solutions to certain systems of linear equations over the reals and give a complete characterization of when the index of Siegel grids is finite.

The results on stability are joint work with Andreas Alpers, that on Siegel grids are joint work with Barbara Langfeld.

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